Gearing up for the Common Core State Standards in Mathematics

Five initial domains for professional development in Grades K–8

Supported by
The Institute for Mathematics & Education, University of Arizona
The Center for Science, Mathematics & Computer Education, University of Nebraska
The Institute for Research on Mathematics and Science Education, Michigan State University

Introduction

The policy workshop *Gearing Up for the Common Core State Standards in Mathematics*, held at the Westward Look resort in Tucson, Arizona on 1–2 April 2011, convened representatives from state departments of education and school districts,[1] the K–12 community, mathematicians, mathematics educators, professional development deliverers, and policy organizations[2] to develop recommendations for the initial domains of professional development on the Common Core State Standards in Mathematics. Participants studied both K–8 and high school standards. In addition to recommending that all professional development incorporate the Standards for Mathematical Practice, this report outlines five recommended domains for initial professional development efforts in K–8:

- Grades K–2, Counting and Cardinality and Number and Operations in Base Ten

[1] Arizona, California, Georgia, Idaho, Indiana, Kentucky, Louisiana, Michigan, Nebraska, Vermont

[2] Council of Chief State School Officers, National Council of Supervisors of Mathematics, Science Foundation Arizona

- Grades K–5 Operations and Algebraic Thinking
- Grades 3–5 Number and Operations—Fractions
- Grades 6–7 Ratios and Proportional Reasoning
- Grade 8 Geometry

These domains were chosen either because they are foundational to what comes later, or because they contain an approach that is likely to be novel for teachers. They do not represent "priority standards" in the curriculum. Indeed, since the standards describe a coherent structure to which all parts contribute, the isolation of "priority standards" is not recommended. Rather, they are recommended starting points for professional development to implement the Common Core..

Each of the five sections in this report gives a rationale for the choice of domain, a brief description of where to start in the domain, and connections with other content standards and practice standards. Standards in the margins are at key confluences or sources in the flow of the standards across grade level, and provide useful starting points for discussions about the design of professional development. The margin also displays the grade level critical areas from CCSS, setting the displayed in context. The displayed standards are not intended to be the sole focus of professional development. They are often either key "understand" standards, or standards describing in some detail a central concept or skill.

Resources for implementing the standards

More detailed descriptions of the domains targeted here may be found in the standards progressions being written by the Progressions Project, `ime.math.arizona.edu/progressions/`. The standards themselves are displayed in a flexible on-line format, viewable by grade level and by domain, at the Illustrative Mathematics Project website, `http://illustrativemathematics.org`. The Illustrative Mathematics Project will also be collecting sample tasks and problems to illustrate the standards. Other tools supporting implementation of the standards are being developed by NCTM, NCSM, AMTE, and ASSM. See `http://commoncoretools.wordpress.com` for announcements about these and other projects.

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We gratefully acknowledge the assistance of Aubrey Neihaus and Teresa Stovall in organizing the workshop.

K–2: Counting and Cardinality & Number and Operation in Base Ten

Why this is a target for professional development

The emphasis on connecting counting to cardinality in Kindergarten, and the emphasis on *understanding* place value in K - 2, are new for many teachers and students. Learning to count is a complex mental activity that requires staying connected to objects that are being counted, moving from the objects to the numbers and from counting back to the objects. Understanding place value means not only being able to identify which place in a multi-digit number is called the 1s place, which the 10s place, and so on, but also understanding that each digit represents an amount of 1s, 10s, or 100s, and that the number is expressed as a sum of these amounts.

Suggested starting points

The relationship between number and quantity

The standards in the cluster *Count to tell the number of objects* emphasize the importance of developing a quantitative meaning for the symbols used.[K.CC.4]

Understand place value

The Kindergarten NBT standard starts a thread which is continued in Grades 1 and 2. In Kindergarten, students decompose teen numbers into 10 ones and some further ones;[K.NBT.1] and in Grades 1 and 2, they move to working with base ten units—ones and tens in Grade 1,[1.NBT.2] and ones, tens, and hundreds in Grade 2.[2.NBT.1] This thread should emphasize bundling of base ten units into higher units.

Another thread, starting in Grade 1,[1.NBT.4] moves to towards fluency with 2-digit addition and subtraction[2.NBT.5,2.NBT.6] and solving word problems involving numbers up to three digits.[2.NBT.7] Visual models and math drawings support an understanding of the base ten system and the properties of operations, and help students explain why addition and subtraction strategies work.[2.NBT.9]

Connections with other standards

Understanding multi-digit computation procedures depends not only on the understanding of base-ten decomposition of numbers as described in this domain, but also on an understanding the behavior of operations and understanding, as described in the Operations and Algebraic Thinking domain. For example, many strategies for adding multi-digit numbers involve decomposing the addends into parts and changing the order in which the parts are added. The domain of

CCSS Critical Areas Grades K–2

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

1.NBT.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.

Counting and Cardinality is a context in which students have the opportunity to work the mathematical practices of Reasoning quantitatively and abstractly (MP2) and Attending to precision (MP6); the domain of Number and Operations in Base Ten provides opportunities for work on Constructing viable arguments and critiquing the reasoning of others (MP3) and Looking for and expressing regularity in repeated reasoning (MP8).

K–5: Operations and Algebraic Thinking

Why this is a target for professional development

Algebraic thinking in the elementary grades involves understanding the structure of the number system, examining the behavior of the operations, and articulating and proving generalizations. Key to such algebraic thinking is developing representations of the operations using objects, drawings, and/or story contexts and keeping these connected to interpretation of symbols.

Eventually students learn to function in the abstract, but they retain the capacity to show how ideas can be represented (e.g. pictures, number lines, manipulatives, physical contexts). Such manipulatives or pictures are not merely "crutches," but are essential tools for thinking.

Teachers must become familiar with the kinds of actions represented by each of the operations, the variety of representations, and the affordances and limitations of different representations. Appropriate representations provide access to the reasoning that underlies methods of calculation.

Suggested starting points

Each operation models a variety of situations

Addition and subtraction can model situations involving joining, separating, and comparing,[1.OA.1,2.OA.1] and multiplication and division can model situations involving multiple groups, splitting, shrinking and stretching, counting objects in rectangular arrays, counting combinations, comparing.[3.OA.3,4.OA.1,4.OA.2] In many interpretations of multiplication (and in contrast to addition and subtraction) the numbers are associated with different units: e.g., 3×4 might model 3 *bags*, with 4 *donuts per bag*. Such multiplication problems correspond to two types of division problem: partitioning into groups of a given size and partitioning into a given number of groups.[3.OA.1,3.OA.2]

In developing broadly based conceptions of the situations modeled by operations, it is important to become familiar with various representations of operations, including arrangements of objects, pictures, story contexts, and number lines.[K.OA.1] Arrays and rectangle representations of multiplication are particularly useful.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS Critical Areas Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Identify and apply generalizations about the behaviors of the operations

In primary grades, students explore the idea that any subtraction problem can be seen as a missing addend problem.[1.OA.4] In later grades, students learn that any division problem can be seen as a missing factor problem.[3.OA.6] Thus, although students initially have many ways of seeing each of the operations, they move toward the idea that subtraction is a derived notion from addition and that division is a derived notion from multiplication.

1.OA.4 Understand subtraction as an unknown-addend problem.

3.OA.6 Understand division as an unknown-factor problem.

Familiarity with generalizations about the behavior of the operations supports the learning of basic facts. That is, students can use what they know about the operations to derive facts from those that are easier to remember or to figure out: $2+7$ must equal 9 because $7+2=9$; $5+6$ must be 1 more than $5+5$; 6×8 has one more group of 8 than 5×8.[1.OA.3,3.OA.5]

Among the generalizations to be explored in the elementary grades are the properties of the operations, though it is not important for students to be able to name the properties. For example, students need not associate the first idea expressed above with the term "commutative property of addition"; nor is it necessary that they recognize the second idea as a special case of the associative property of addition. Whatever language students use to identify their generalizations, it is important that they recognize their continued re-occurrence in computation; the properties of operations form an important tie between the Operations and Algebraic Thinking domain and the Number and Operations in Base Ten domain.[2.NBT.5, 3.NBT.2, 4.NBT.5, 5.NBT.6]

1.OA.3 Apply properties of operations as strategies to add and subtract.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

When students learn about different operations, they frequently remark on patterns they notice. For example, they might use their own words to express the idea that the addends of an addition expression can be exchanged and the total remains the same. Or they might notice that when 1 is added to an addend, then the total increases by 1. Classroom work on noticing, exploring, and articulating such generalizations[3.OA.9] refers back to the various representations of the operations to see how and why these generalizations hold.

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Attention to generalizations allows students to identify differences among the operations. For example, unlike addition, changing the order of terms in a subtraction expression *does* change the result of the subtraction. Also in contrast to addition, in the realm of whole numbers, adding 1 to a factor of a multiplication expression generally changes the product by more than 1.

Connections with other standards

The content of Operations and Algebraic Thinking connects to other domains at the elementary level. Understanding the behavior of the operations for whole numbers is essential to making sense of fractions and decimals. With an understanding of the variety of contexts for using different operations, students can solve problems involving measurement. In the domain of Geometry, representations of multiplication support students' understanding of rectangles and area.

The content of this domain also supports student learning in later grades. Understanding the distinction between additive and multiplicative contexts is essential to engaging with Ratios and Proportional Relationships. Expressions and Equations in grades 6–8 and Seeing Structure in Expressions at the high school level are natural extensions of Operations and Algebraic Thinking in grades K–5. Throughout the grades, students work back and forth between symbols and a variety of representations or contexts from which expressions might be drawn. They need to be able to look at the components of an expression and think about the behavior of the operations involved to see how the value of an expression changes as the value of the variables change.

The domain of Operations and Algebraic Thinking is a context in which students have an opportunity to work on such mathematical practices as Reasoning abstractly and quantitatively (MP2), Constructing viable arguments and critiquing the reasoning of others (MP3), Looking for and making use of structure (MP7), and Looking for and expressing regularity in repeated reasoning (MP8).

3–5: Number and Operations—Fractions

Why this is a target for professional development

The Common Core describes the development of fractions and operations on fractions in a careful sequence over Grades 3–5 (partially extending into the Grade 6 Number System domain), which may be unfamiliar to many teachers and teacher educators. Students represent fractions in multiple ways with an emphasis on viewing fractions as numbers on the number line. In order to foster this understanding, teachers must give students opportunities to reason both abstractly and quantitatively about fractions (MP2).

Suggested starting points

Attention to the meaning of fractions as numbers

Students begin to learn about fractions as numbers by working with unit fractions.[3.NF.2] This early work with fractions parallels students' initial learning about arithmetic with whole numbers; for example, students learn that $3/5 = 1/5 + 1/5 + 1/5$ and that $4/7 = 4 \times 1/7$.[4.NF.3b, 4.NF.4a] Teachers should be encouraged to use visual models such as the number line to reinforce the idea that fractions are part of the number system and to establish basic arithmetic properties of fractions. The emphasis on unit fractions as an initial

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSS Critical Areas Grades 4–5

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

[3.NF.2] Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

[4.NF.3b] Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

[4.NF.4a] Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a Understand a fraction a/b as a multiple of $1/b$.

stepping stone to doing arithmetic with fractions is different from many current professional development programs.

Attention to the whole

When teaching students to reason about fractions as representations of parts of objects, teachers will need to continually reinforce the importance of paying attention to the whole to which each fraction refers. Fractions can only be compared or added when they refer to the same whole;[3.NF.3d] students should be given opportunities to work on problems in which "the whole" has several possible interpretations and observe what happens under each interpretation. It is important to support teachers in creating rich tasks that help students develop problem solving strategies such as the use of visual fraction models, which allow students to visualize the relationship between a fraction and the whole.[4.NF.3d]

Extending the meaning of the operations to fractions

Students should eventually attain procedural fluency in performing arithmetic operations on fractions. Students need to be encouraged to use their understanding about the meaning of arithmetic operations to extend these operations to fractions,[5.NF.4a,5.NF.7] culminating in procedures such as invert and multiply for dividing fractions. The increased emphasis on understanding the logic underlying algorithms for operations should be a focus of future professional development.

Connections with other domains

Success in this domain depends in large part on the foundation that has been laid in Operations and Algebraic Thinking. Not only do students need to understand models for operations in order to be able to extend these operations to fractions; they should also be able to use their knowledge of relationships among operations to devise strategies for performing operations. Teachers should be encouraged to highlight these relationships whenever possible.

The Fractions domain in grades 3 through 5 sets the groundwork for work on Ratios and Proportional Relationships in grades 6 through 8. Teachers can make connections to this domain by giving students opportunities to see that a given fraction can represent different amounts, depending on the choice of whole. This introduces students to strategies such as scaling that are useful in analyzing proportional relationships.

The domain of Number and Operations—Fractions is a context in which students have an opportunity to work on such mathematical practices as Reasoning abstractly and quantitatively (MP2), Attending to precision (MP6), and Looking for and making use of structure (MP7).

3.NF.3d Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- d Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, =, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

4.NF.3d Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

- d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

5.NF.4a Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

- a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
- b Interpret division of a whole number by a unit fraction, and compute such quotients.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

6–7: Ratios and Proportional Relationships

Why this is a target for professional development

The language of the standards is different from what teachers might be used to. Standards calling for understanding rather than solving a specific type of problem may be challenging to assess in a classroom situation. Teachers should be encouraged to emphasize problems in which students explore ratios and rates in real-world contexts.

The idea of a proportional relationship is a common thread that emerges in algebra, geometry, and statistics. In many modeling problems, it is necessary to identify a proportional relationship that arises in a real-world situation and analyze or extend that relationship. Grade 6 students use "ratio and rate reasoning" to solve problems in which a multiplicative comparison between two quantities is called for ("how many times as much", "at what rate"), and Grade 7 students study "proportional relationships", in which two varying quantities are related by a constant rate.[6.RP.1][6.RP.2]

Critical content areas for initial professional development

Proportional relationships as a unifying principle

In Grade 5, students prepare for work on ratios and proportional relationships by interpreting multiplication as scaling.[5.NF.5a] In Grade 6, it is important that teachers model reasoning about proportional relationships using ideas such as ratio, rate, and scaling, rather than simply executing rote algebraic techniques (such as "set up a proportion and cross multiply").[6.RP.1, 6.RP.2] In Grade 7, teachers should ask students to decide whether a given relationship is proportional,[7.RP.2a] and ask students to explore in a given situation whether a multiplicative comparison or an additive comparison is the most appropriate way to compare two quantities.

Modeling and applications

Students apply the ideas of ratio, rate, and proportional relationship to modeling problems, in which these ideas have real-world meanings.[6.RP.3] Flexibility in problem-solving strategies should be encouraged; for example, students can express a relationship by scaling up to a whole number ratio when it is appropriate and efficient rather than always having to scale down to a unit rate.

Connections with other domains

The domain of ratios and proportional relationships connects fractions to functions. Students in Grades 6 and 7 should have prior experience using fractions in modeling problems; one key goal of work in this domain is to introduce the idea that a relationship

CCSS Critical Areas Grades 6–7

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

5.NF.5a Interpret multiplication as scaling (resizing), by:

a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

7.RP.2a Recognize and represent proportional relationships between quantities.

a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b Solve unit rate problems including those involving unit pricing and constant speed.

c Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

between two quantities involving a constant ratio or rate can be represented algebraically. This skill is a prerequisite for working with similarity relationships in geometry and for making inferences about a population in statistics.

The domain of Ratios and Proportional Relationships is a context in which students have an opportunity to work on such mathematical practices as Making sense of problems and persevering in solving them (MP1), Reasoning abstractly and quantitatively (MP2), Modeling with mathematics (MP4), and Using appropriate tools strategically (MP5).

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

8: Geometry

CCSS Critical Areas Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bi-variate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Why this is a target for professional development

Geometry in the Common Core State Standards is based on transformations, an approach that is significantly different from previous state standards. This is a change for students, teachers, and teachers of teachers. Challenges include attention to precision and language about transformations. Professional development will need to support teachers to establish a vision for the mathematics classroom (videos or other means of seeing it happening) related to this domain of instruction. Analysis of student work in this domain will also be essential for developing this vision.

Suggested starting points

Describing and defining rotations, reflections translations, and dilations

Professional development should support teachers in further developing their knowledge of transformations and their properties.[8.G.1] Students in Grade 8 get hands on experience with rotations, reflections, translations, and dilations, observing their effects on two-dimensional figures on the coordinate plane.[8.G.3] This prepares for work in high school on formulating precise definitions of these transformations.

[8.G.1] Verify experimentally the properties of rotations, reflections, and translations:

a Lines are taken to lines, and line segments to line segments of the same length.

b Angles are taken to angles of the same measure.

c Parallel lines are taken to parallel lines.

[8.G.3] Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Understanding congruence and similarity

The transformational approach to congruence and similarity is likely unfamiliar to many middle grades teachers. In high school students take the properties of operations as assumed, and from them derive the standard criteria for congruence and similarity of triangles. In preparation for this, middle school students develop an understanding of congruence and similarity in terms of transformations.[8.G.2,8.G.4] Careful attention by professional development providers to differences between previous approaches to the topic and the transfor-

[8.G.2] Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[8.G.4] Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

mational approach, and effective communication of these differences to teachers, is essential.

Connections with other domains

The transformational approach to congruence and similarity should be connected to work within the coordinate plane. Within the geometry standards, teachers will need to guide their students in transforming two-dimensional figures using coordinates. In addition, similarity and proportional relationships are connected to the expression and equation standards.[8.EE.6]

The domain of Geometry is a context in which students have an opportunity to work on such mathematical practices as Constructing viable arguments and critiquing the reasoning of others (MP3), Using appropriate tools strategically (MP5), and Looking for and making use of structure (MP7).

[8.EE.6] Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.